LSTM Model and Stock Price Prediction

Stock Price Equation

The price of a stock $P(t)$ at discrete time $t \in \{t_1, t_2, t_3, \ldots\}$ is given by:

$$P(t) = P(t-1) + F_{\text{macro}}(t) + F_{\text{micro}}(t) + F_{\text{technical}}(t) + F_{\text{noise}}(t)$$

- $P(t-1)$: Price of the stock at the previous time step.
- $F_{\text{macro}}(t)$: Macro-level influences.
- $F_{\text{micro}}(t)$: Micro-level influences.
- $F_{\text{technical}}(t)$: Technical analysis factors.
- $F_{\text{noise}}(t)$: Stochastic noise term.

Macro Influences

$$F_{\text{macro}}(t) = \alpha_1 G(t) + \alpha_2 I(t) + \alpha_3 R(t)$$

- α_i: Weights determining the strength of each factor.
- $G(t)$: GDP growth/market sentiment, modeled as:

$$G(t) = y \sin\left(\frac{2\pi t}{T_B}\right) + N_2 Z_2(t)$$

- $I(t)$: Inflation rate, modeled as:

$$I(t) = \Theta e^{-\lambda_0 t} + N_2 Z_2(t)$$

- $R(t)$: Risk-free interest rate:

$$R(t) = r_0 + N_3 Z_3(t)$$

Micro Influences

$$F_{\text{micro}}(t) = \beta_1 E(t) + \beta_2 S(t) + \beta_3 C(t)$$

- $E(t)$: Earnings per share, $E(t) = E_0 e^{\mu t}\left[1 + \sin\left(\frac{\pi t}{T_E}\right)\right]$
- $S(t)$: Scale growth rates:

$$S(t) = \frac{\text{Max scale level}}{1 + e^{-K_0(t-t_0)}} + N_5 Z_5(t)$$

- $C(t)$: Competition index:

$$C(t) = \frac{1}{t + \text{Season growth cycle}} + N_6 Z_6(t)$$

Technical Factors

$$F_{\text{technical}}(t) = \delta_1 M(t) + \delta_2 V(t)$$

- $M(t)$: Momentum, $M(t) = P(t+1) - P(t+5)$
- $V(t)$: Volatility:

$$V(t) = \sqrt{\frac{1}{W}\sum_{i=1}^{N}[P(t-i) - \overline{P}(t)]^2}$$

where $\overline{P}(t) = \frac{1}{N}\sum_{i=1}^{N} P(t-i)$.

Noise Term

$$F_{\text{noise}}(t) = \sigma Z(t)$$

- $\sigma Z(t)$: Noise term, where $Z(t) \sim N(0,1)$.

LSTM Architecture

1. Feature vector $X(t)$:

$$X(t) = \begin{bmatrix} P(t-2) \\ P(t-1) \\ P(t) \\ G(t) \\ E(t) \\ S(t) \\ C(t) \\ M(t) \\ V(t) \end{bmatrix}$$

2. LSTM components:

$$f(t) = \sigma(W_f X(t) + U_f h(t-1) + b_f)$$

$$i(t) = \sigma(W_i X(t) + U_i h(t-1) + b_i)$$

$$\tilde{C}(t) = \tanh(W_c X(t) + U_c h(t-1) + b_c)$$

$$C(t) = f(t) \cdot C(t-1) + i(t) \cdot \tilde{C}(t)$$

$$o(t) = \sigma(W_o X(t) + U_o h(t-1) + b_o)$$

$$h(t) = o(t) \cdot \tanh(C(t))$$

Loss Function

$$\text{MSE} = \frac{1}{T} \sum_{t=1}^{T} [P(t) - \hat{P}(t)]^2$$

$$\text{MAE} = \frac{1}{T} \sum_{t=1}^{T} |P(t) - \hat{P}(t)|$$